

Core Mathematics C3 Advanced Level

For Edexcel

Paper I

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

The booklet 'Mathematical Formulae and Statistical Tables', available from Edexcel, may be used.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner.
Answers without working may gain no credit.

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1. The function f is defined by

$$f: x \mapsto \frac{1 - 2x}{2 - x}, \quad x \in \mathbb{R}, \quad x \neq 2.$$

(a) Prove that $f^{-1}(x) = f(x)$ for all $x \in \mathbb{R}, x \neq 2$. (3)

(b) Hence find, in terms of k , $ff(k^2)$, where $k^2 \neq 2$. (2)

2. The function g is given by

$$g: x \mapsto \ln |4x - 12|, \quad x \in \mathbb{R}, \quad x \neq 3. \quad (3)$$

(a) Sketch the graph of $y = g(x)$.

(b) Find the exact coordinates of all the points at which the curve $y = g(x)$ meets the coordinate axes. (3)

3. $f(x) = x - \frac{1}{x-2} + \frac{5}{x^2+x-6}, \quad x \in \mathbb{R}, \quad x > 2.$

(a) Show that $f(x) = \frac{x^2 + 3x - 1}{x + 3}$. (5)

(b) Solve the equation

$$f'(x) = \frac{26}{25}. \quad (5)$$

4. (a) Given $y = \frac{e^{5x}}{x}$, find $\frac{dy}{dx}$ and the value of x for which $\frac{dy}{dx} = 0$. (4)

(b) (i) Given $x = \sin^2 3y$, find $\frac{dx}{dy}$ in terms of y . (3)

(ii) Evaluate $\frac{dy}{dx}$ for $y = \frac{\pi}{12}$. (3)

5. (a) Express

$$2.5 \sin 2x + 6 \cos 2x \quad \text{in the form}$$

$R \sin(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving your values of R and α to 3 decimal places where appropriate. (4)

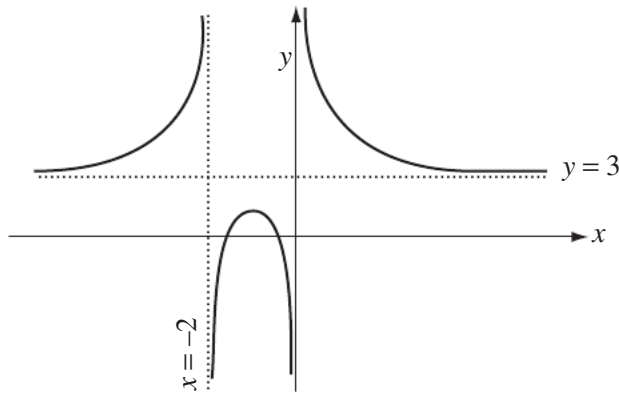
(b) Express $5 \sin x \cos x - 12 \sin^2 x$ in the form

$$a \cos 2x + b \sin 2x + c, \quad \text{where } a, b \text{ and } c \text{ are constants to be found.} \quad (4)$$

(c) Hence, using your answer to part (a), deduce the maximum value of

$$5 \sin x \cos x - 12 \sin^2 x. \quad (2)$$

6.



The diagram shows the sketch of part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$, $x \neq 0$, $x \neq -2$.

The curve has a maximum at $(-1, 2)$.

The lines $y = 3$, $x = -2$ and the y -axis are asymptotes to the curve as shown.

On separate diagrams sketch the graphs of

(a) $y = |f(x)|$ (3)

(b) $y = f(2x)$ (3)

(c) $y = f(x - 1) - 2$ (4)

In each case state the equations of the new asymptotes and the coordinates of the turning points.

(d) Solve the equation $f(x - 1) - 2 = 0$ (2)

7. The curve C has the equation $y = f(x)$ where

$$f(x) = \frac{1}{2} \ln x + \frac{1}{x^2}, \quad x > 0.$$

P is a stationary point on C .

(a) Calculate the x -coordinate of P . (4)

(b) Show that the y -coordinate of P can be expressed in the form $k^{-1} \ln k + k^{-2}$, where k is a constant to be found. (2)

The point Q on C has x -coordinate 1.

(c) Show that the equation to the normal at Q can be written as

$$Ay + Bx + C = 0$$

where A , B and C are integers to be found. (3)

8. A cup of tea, initially at boiling point, cools according to Newton's law of cooling so that after t minutes its temperature, $T^\circ C$, is given by

$$T = 15 + 85e^{-\frac{t}{8}}.$$

(a) Sketch the graph of T against t . (3)

(b) What is the temperature of the tea after 4 minutes? (2)

(c) How long does it take the tea to cool to $40^\circ C$? (3)

(d) Find $\frac{dT}{dt}$ and hence find the value of T at which the temperature is decreasing at the rate of $1.7^\circ C$ per minute. (4)

(e) However long the cup of tea is left to cool down, it never falls below a certain temperature. What temperature is that? (1)

END

TOTAL 75 MARKS